Title

Chi Feng Lina and Chunshien Lib

Information Management. National Central University, National Central University, Taoyuan, TAIWAN

aEmail: j8888888871@yahoo.tw

bEmail: [jamesli@mgt.ncu.edu.tw](mailto:jamesli@mgt.ncu.edu.tw) (corresponding author)

*Abstract*—Stock volatility is a time series of data. Time series prediction is an important research topic, and artificial intelligence is currently being widely used in this topic, such as: neuro-fuzzy systems. This paper proposes a complex neuro-fuzzy system and applies it to multi-target time series prediction. This model has multiple complex-valued outputs. For each complex-valued output, the real part and the imaginary part can be predicted separately for two different real-valued targets. For the feature selection, this study uses multi-target feature selection to screen out the features that are beneficial to all targets, and use them as a model input to reduce the overall model computational burden and improve the efficiency of data application. In terms of model, a multi-layer neural network is constructed by input layer, complex fuzzy sets layer, premise neural layer, Takagi-Sugeno neural layer and output layer. In terms of parameter learning, we use the divide-and-conquer principle when training model. The parameters of the complex fuzzy sets layer are optimized using different algorithms, such as particle swarm optimization (PSO) and artificial bee colony optimization (ABCO); The parameters of the T-S neural layer are optimized using recursive least-squares estimation (RLSE); other neural layers have no parameters to be optimized. In terms of experiments, we designed three experiments to test the performance of the model, combining the PSO-RLSE and ABCO-RLSE experimental results with the investment strategy, the calculated model profits are compared with each other and compared with different literature methods. This study proposes a new investment strategy, compared with the past investment methods, and through the result of the performance comparison and the profit comparison. The multi-target prediction method proposed in this paper shows excellent performance and investment effect.

Keywords—Time series; Complex fuzzy set; Complex neuro-fuzzy system; Artificial bee colony optimization; Particle swarm optimization; RLSE;

# Introduction

Time series data is a sequence of data arranged in order of time. For example, the stock price and exchange rate are all time series data. The time series can be regarded as data observed in discrete time order, and can be analyzed by mathematical methods or statistical models, which are commonly used method today. With the rapid development of the Internet and the advancement of technology, the amount of information has increased rapidly. Using artificial intelligence, such as machine learning or data mining, the hidden relationships or rules hidden behind the data can be found, which cannot be found by human intelligence.

The prediction of time series data is widely used in various fields, and there is many research, such as financial exchange rate, stock ups and downs, etc.; In addition to financial field, energy consumption, disease prediction and so on can be used in the allocation of the resources, which helps countries or institutions to make effective resource allocation. Among the above-mentioned fields, the most relevant to us is the financial forecast. How to effectively use the money investment to make profits is a topic worth exploring. Time series data forecasting in the financial economy is the most difficult because it includes many influence factors, such as company situations, global situation, and overall economic environment. Many factors make financial time series highly variable. Therefore, if there is a model with good prediction performance which has effective input data and correct investment strategy, can help investors get profits from it.

In the prediction of finance, the neural network is the most widely used method. Since 2005, there have been many research outputs on financial issues in the real world. For example, Zhi-Bin et al. [43] used the adaptive neuro-fuzzy inference system (ANFIS) and the artificial neural network (ANN) to predict annual excess returns and compare performance. He found that both predictions are very good; In 2006, Patel et al. [30] used ANFIS, fuzzy inference system and ANFIS to assist investors in making economic decisions; Yao [40] proposed a method in 2007 to deal with foreign currency trading strategies, profiting from forecasting exchange rates; Li et al. [25] used group intelligence and self-organized neuro-fuzzy system (NFS) to predict the exchange rate between RMB and US dollars; In addition to the exchange rate, stock forecasts also have many research literatures. For example, Abbasi used ANFIS in 2008 with four independent variables, trading volume, price-earnings ratio and earning per share forecasting stock closing price [1]; in 2017, Koijen et al. [22] predicted each other's data through stocks and bonds; Pan et al. [30] compared short-term and long-term leverage by predicting stock volatility.

In the stock market, there are several famous theories, one of which is the random walk theory proposed by Kendall et al. [18] in 1953, which means that the stock price changes are independent, and there is no regularity or period; Its extension is efficient market hypothesis (EMH) [11], which was proposed by scholar Fama in 1970, this theory holds that the transaction price is acceptable to both the buyer and the seller, and that all participants in the market can obtain information without compensation, so the information collected by the investor cannot make excessive profits. But other experts believe that stocks are predictable, so they research continuously. In 1990, Kimoto et al. [21] used the back propagation neural network (BPNN) to match the two indicators to predict the rise and fall of the Nikkei and the timing of buying and selling; In 1999, Yao et al. [39] used a number of technical indicators to predict the stock market by using back propagation neural networks; Kim et al. [20] added the genetic algorithm (GA) to the neural network in 2000, which not only improved the learning speed of the neural network, but also reduced the complexity of the feature space. The back propagation neural network appears to be more excellent, and it is found that the nonlinear neural network has better predictive ability; In 2012, Wei [37] proposed an ANFIS based on empirical mode decomposition (EMD), and calculated the profit with investment strategy. In virtual investment, there is a good profit-making effect; In the same year, Li et al. [26] used the autoregressive integrated moving average (ARIMA) model combined with the neuro-fuzzy system (NFS) to predict the closing price of stocks. The following year, Li et al. [27] used a complex neuro-fuzzy system (CNFS) to simultaneously predict dual targets.

As can be seen from the above, the NFS is very useful for predicting financial aspects, and a well-trained model performance is much more accurate. Therefore, this study will use the NFS to combine with other artificial intelligence techniques to predict multiple targets. In the process of model training, problems such as prediction bias or long-term model operation often occur. Therefore, how to select effective input data and control the size of the model is the main topic of most research. In the process of forecasting, the parameters need to be optimized to increase performance. Many machine learning algorithms are currently used to find the optimal solution of parameters. Such as particle swarm optimization (PSO) [19], artificial bee colony optimization (ABCO) [17], gene algorithm (GA) [10] and so on. At present, most of the literatures are based on the Takagi-Sugeno fuzzy system [35]. Therefore, there are many parameters that need to be optimized, which will make the algorithm parameters too large, which causes the effect less than expected. Besides, it makes the algorithm convergence too slow and fall into the local minimum.

In this study, a hybrid algorithm is used to optimize the parameters. The algorithm PSO and ABCO are respectively combined to recursive least squares estimation (RLSE) algorithm [16] to form PSO-RLSE [23] and ABCO-RLSE. Different algorithms are used to optimize the parameters of different parts by the divide-and-conquer method, which reduces the probability that machine learning will fall into the local minimum when seeking the solution. As mentioned above, stock volatility is formed by many factors. Therefore, this study will select features from multiple stocks, and predict multiple targets at the same time, extracting content that is beneficial to each target from a large amount of data. In order to improve the prediction effect, the neural network system is combined with the fuzzy rule of Takagi-Sugeno (T-S) type to form a neuro-fuzzy model. And the parameters of the neuro-fuzzy model are divided into two parts, namely the if-part parameters and the then-part parameters.

Traditionally, the design of fuzzy sets is based on mathematic functions to establish fuzzy sets [42]. After data input, the membership degree which is between 0 and 1 can be obtained. In 2002, Ramot proposed complex fuzzy sets (CFSs) [31], adding an imaginary part to the original membership degree. The membership degree of CFSs is that traditional one-dimensional space projecting to the unit complex disc which the axis x is real part and the axis y is imaginary part. This makes it have a richer information than the original real-type membership function.

This study proposes a hybrid algorithm, PSO-RLSE and ABCO-RLSE, to optimize model parameters. There are two part parameters in the model, namely the if-part parameters and the then-part parameters. PSO and ABCO are responsible for optimizing the if-part parameters, and RLSE is to optimize the then-part parameters, to reduce the search dimension of each algorithm.

In this study, firstly, we use the concept of Shannon information entropy [34] to analyze data and make multi-target feature selection [28]. By calculating the amount of information provided by the original data to the target as a standard, and adding the calculation of redundant information, selecting the data that has a good influence on the target as the basis for input to the model can reduce the data complexity and the computation cost. Then, through the PSO-RLSE and ABCO-RLSE algorithms update the parameters of the if-part and the parameters of the then-part. Finally, the results are combined with the investment strategy to calculate the profit brought by the model.

This study is divided into six sections. The section I is the introduction, introducing the background, motivation and purpose of the research, and summarizing the methods and processes used in this study. The section II introduces and summarizes the theoretical methods, including some methods of past feature selection. In addition, the origin of fuzzy sets and the concept of complex fuzzy sets will be introduced. Then introduce the theory of neural network, and related research, and finally mention the widely used neuro-fuzzy system. The section III is the system design and architecture, an overview of multi-target feature selection, and a method of machine learning to determine the size of the model. The introduction of the various neural layers in a complex neuro-fuzzy network is also described in this section. Then the algorithm of parameter learning will be described in detail, including PSO, ABCO, and RLSE, and the process of hybrid algorithm in training phase. Finally, it will mention the operation and concepts of past investment strategies, as well as introduce the new investment strategy methods and new method of calculating profits. The section IV is the experiment and the results. In this study, three experiments were used to study the time series prediction. Experiment 1 predict single target, the second experiment simultaneously forecasts two targets, the third experiment is a forecast for four targets. The above experiments will make two comparisons, one is the performance comparison of the two different hybrid algorithms in the study, and the profit comparison with the investment strategy proposed in the past and the investment strategy proposed in this study. In addition, the performance comparison with the methods proposed in the past literature will be carried out to confirm the pros and cons of the research method. The section V discusses the results of the experiment. The sections VI is the conclusion of this study and the future research direction.

# Literature Review

## Feature Selection

The data calculation model is constructed through machine learning or data mining, such as classification, regression, and clustering. There are two purposes for preprocessing data. One is to reduce the size of the data set, making the analysis more effective. The other is to select the appropriate data for a specific data set. For today's big data, the former is very important for the entire analysis method, so feature selection plays an indispensable role in many fields such as data mining, network anomaly detection, text classification, genetic analysis and so on. The reduction of feature sets is based on the relevance of the features and their redundancy to the target [41]. Feature selection methods can be classified into several [14], the most common are the filter approach [12], the wrapper approach [2], the embedded method [12], and the Hybrid approach [15]. The following is a brief description of each method.

The filter approach was proposed by Guyon [12] in 2003. This method first selects the feature set of the data set, then evaluates and scores the selected feature subsets, and finally sets the threshold value. The process of feature selection is independent of models, which has the advantage of being faster than the wrapper approach. In 1997, Blum proposed wrapper approach [2], the biggest difference from the filter approach is the selection of the feature sub-sets. The scoring criteria are determined by the results of the model. So in the selection process, it will continuously run model computation, which have long calculation time and complicated calculation. For some specific classifiers, the wrapper approach has high classification accuracy. The embedded approach [12] combines feature selection with the training of the model. Therefore, feature selection is also completed after the model training ends. The feature selection time and model computation are between the above two methods. The hybrid approach [15] combines the concepts of the filter approach and the wrapper approach, selects appropriate feature subsets through the filter approach, and then uses these feature subsets in model to select the final feature. This method improves the accuracy of the filter approach and reduces the computational time of the wrapper method, combining the advantages of both.

As mentioned above, the reduction of feature sets is based on the relevance and redundancy of features. Therefore, this study uses concept of entropy to calculate the amount of information provided by features. The word entropy was first proposed by the German physicist Rudolph Clausius in 1854 [8], it is a measure of the disorder of the physical system. When the entropy value is higher, the degree of disorder is higher. Regarding the disorder of information, Shannon proposed the theory of Shannon information entropy [34]. If the randomness of information is higher, the information entropy value will be higher. For a random variable , information entropy The definition is as follows.

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where, ， is the entropy of the random variable ; is the probability of the event ; is regarded as the disorder information of the . It can be found that the greater the probability of event occurrence is, the smaller the information entropy value is. Conversely, if the probability of an event tends to average, the entropy will approach the maximum. As mentioned before, the larger the entropy value, the higher the randomness of the information provided. This study is based on the theory of Shannon information entropy [34] to make multi-target feature selection [28]. This method takes into account the information and the redundant information. Finally, the user's settings are used to filter out several features as input to the model.

## Complex Fuzzy Sets

The concept of the crisp set is only “belonging” or “non-belonging”, but this concept cannot be applied to the real world. For example, if the temperature is greater than 30 degrees, it is called “heat”, but 29.9 degrees is not much different from 30 degrees but it is not “hot”. This situation conflicts with human thinking. In 1965, Zadeh proposed the concept of fuzzy sets [42]. By this concept, we can calculate the membership degree of the elements belonging to the set through a membership function, and the membership degree is between 0 and 1. The higher membership degree, the higher the level to which the element belongs to the collection. Afterward, Buckley proposed the complex number [3-5]. In 2002, Ramot et al. [32] further proposed complex fuzzy sets (CFSs), which means that the complex-valued membership degree can be obtained by a function, which allows the membership degree to be presented in a complex unit disc with a radius of 1. This concept enriches the membership degree.

## Neural Network

Neural network is a technique that mimics the human brain. The human brain contains lots of neurons which main function is to process information and memory. The neurons are connected by "synapses", and the functions of "synapses" are like weights, which control the flow of information. In 1943, McCulloch et al. [29] used an algorithm called threshold logic to simulate the concept of neurons, while a neuron receives information, it will determine whether it is necessary to generate an excitatory response mechanism. Then the psychologist Hebb proposed Hebb's law [13], which means that if the neuron behind synaptic and the neuron in front of synaptic are activated simultaneously, the strength of the connection between the two neurons is increase; This idea was implemented with mathematic model by Rochester et al. [32] in 1956 to create a perceptron to simulate the human brain, which is arguably the ancestor of deep learning. However, the number of neurons is less, and the weight of the transmitted signal is 0 or 1, still cannot handle the XOR problem (non-linear problem), and the computer did not have enough ability to calculate in that time. Until 1975, Werbos [38] proposed back propagation algorithm (BP), which effectively solved the XOR problem and the problem of training multi-layer neural networks. From that time, many other artificial intelligence methods are proposed, such as support vector machine (SVM) [9], Bayesian classifier (Bayesian classifier) [33] and so on. Until recently, due to the rapid development of technology, the speed of computation has increased, and in some areas, neural networks have surpassed humans, neural network was once again heavily studied.

## Complex Neuro-fuzzy system

In the past, many research methods have been proposed such as ARIMA, fuzzy theory, neural network computing, and so on. Among them, the most commonly proposed are neuro-fuzzy systems (NFSs). The neuro-fuzzy system is a fuzzy system, which can be regarded as a combination of neural network theory [29] and fuzzy theory [42]. There are literatures that classify NFSs into three types [36], cooperative NFS, concurrent NFS, and hybrid NFS.

***Cooperative NFSs:*** First, the neural network is used to calculate the required parameters such as rules, and the parameters are brought into the fuzzy system for computation.

***Concurrent NFSs:*** After the data is processed by the neural network, it is regarded as the input of the fuzzy system and the final result is obtained by fuzzy system.

***Hybrid NFSs:*** It is the most widely studied system among the three, which integrates rules and other elements in the fuzzy system into the neural network and corrects the parameters in the system through learning algorithms.

Due to the characteristics of the neuro-fuzzy system, it has a good effect on the prediction of time series. Most of the research on time prediction in this area now uses a neuro-fuzzy system as the model architecture. In this study, a hybrid neural network is used. In order to enrich the information covered by the fuzzy system, this study uses a complex fuzzy set to replace the traditional fuzzy set to form a complex neuro-fuzzy system (CNFS), which allows the model to predict multiple targets simultaneously, the details will be discussed in section III.

# Methodology

This section will explain the method design and model architecture used in the research. In this study, machine learning is used to determine the model structure size. The model is implemented using a CNFS. Different algorithms are used to optimize the parameters of the if-part in the model. The recursive least squares algorithm optimizes the parameters of the then-part. Before the data enters the model, through the multi-target feature selection [28], select the feature data sets that are more effective for all the targets, and reduce the burden of the model. Finally, the results are combined with the investment strategy to make a comparison of different algorithms.

## Complex Fuzzy Sets

The traditional fuzzy set concept [42], the membership degree of an element to a set is one-to-one. A complex fuzzy set can have a richer membership degree. Through this concept, the complex-valued membership degree of a piece of data can be calculated, which makes the model can predict multiple targets at a time. The process of membership calculation is as follows, assuming that there is a complex fuzzy set ,which can be expressed as follows.

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where, is the membership degree of element , denoted as follows.

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where,  is value variable of universal set ; is the amplitude function, it is real-valued between 0 to 1; is the phase function, it is real-valued; is .

This study uses a Gaussian complex fuzzy set, which was proposed by Li et al. [24], is a combination of complex fuzzy set and Gaussian function. Complex Gaussian membership function (cGMF) is denoted as follows.

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where, are input data, center and spread of the fuzzy set, is the phase frequency parameter, this parameter enters the parameter learning to increase the flexibility of the model. is the first order differential of Gussian function, the purpose is to re-use Gaussian original parameters and reduce the complexity of the parameters during the computation. Through the complex Gaussian fuzzy set, a complex degree can be obtained. We can decompose to obtain a membership degree vector and the components are expressed as follows.

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where, is the real part of the value; is the imaginary part of the value; is Gussian function as formula (5); is the first order differential of Gaussian function as formula (6). Through the above disassembly, the membership degree different from the traditional fuzzy set can be obtained without increasing the parameters. This method provides a richer, which is convenient for more applications in the future.

## Structure Learning

Structure learning is to build a more appropriate model architecture through training data. In addition, the results of structural learning will become part of the subsequent parameter learning. In this study, Gaussian fuzzy sets are used, which requires two parameters, center and standard deviation. Therefore, the training data of different input dimensions are clustered by subtractive cluster (SC) algorithm [7]. The clustered group center is combined with the standard deviation of each dimension to form fuzzy sets. The sum of the fuzzy sets of each dimension is the number of the neurons in the first layer. Based on the fuzzy sets of each input dimension, a total of blocks can be formed.

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where, is the number of clusters in the input dimension, which is the number of fuzzy sets. For example, the construction of the area is as follows.

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, where, is the linguistic variable in the input dimension; is the input variable, ; is the fuzzy set of the input linguistic variable in the area, which constructed by Gaussian function as formula (5).



Fig. 1. Fuzzy sets input space. (2-dimensions)

Two input dimensions separately cluster 3 clusters and 9 areas can be obtained, where z-axis is the data density in the area.



Fig. 2. Sum of the data density.

It can be found some areas have high data density, means the area is beneficial for model construction.

In order to reduce the burden of the model computation for improving the efficiency, we select some important areas as neurons in the if-part layer which are the neurons in the second layer. For example, if there are two input dimensions, areas in Fig. 1 can be obtained. Through the data density concept, sprinkle the data into the block, calculate the data density, and accumulate the data density of each area as shown in Fig. 2. Finally, the high data density areas can be selected as the neurons in the second layer. Detail process is as follows.

1. The data density of each block can be obtained from each input dimension and fuzzy set. Taking the block as an example, the data density formula is as follows.

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where, is the data in the dimension; is the fuzzy set of the input dimension in the area.

1. The sum of the data density in the area is denoted as , as follows.

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where, is the number of data. Calculate mean of denote as , standard deviation denoted as .

1. Check each area, if , then . Set the upper and lower bound denoted as and , find the by upper and lower bounds, which are the number of selected areas. If is between the upper and lower bound, set to ; if is smaller than lower bound, set to ; if is bigger than upper bound, set to .
2. Sort and retain the top areas as the neurons in the second layer.

In this study, the number of then-part neurons which are in the fourth layer is same as the number of neurons in the if-part layer. The neuron in the then-part is T–S neuron, which constructed by T–S function, T–S function is as follows.

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where, {} are the parameters of the T–S neuron; is the input.

After structure learning, the number of neurons in the first layer is confirmed, through the area selection, neurons in the second and the forth layer can be obtained for constructing model. The detail of the model will be discussed in the next section.

## Complex Neuro-fuzzy System

This study uses Takagi-Sugeno (T-S) function to construct the system. T-S fuzzy system is first proposed by Takagi and Sugeno in 1985 [35]. It is a composite nonlinear system combined with a series of If-Then fuzzy rules. This study uses the nonlinear cGMF system to combine the linear T-S function to form the nonlinear If-Then rules neural network. If-Then fuzzy rules are like the human experience, so it is more easily understood by humans. In this subsection, the input, calculation and the output in every layer will be discussed.

The model in this study is a hybrid nonlinear system. Training data is denoted as , is t he number of data, is a -by-1 input vector, is the number of input dimensions; is a -by-1 target vector, is the number of complex-valued targets. is the model output.

The model is constructed by T-S fuzzy rules, each rule is combined by if-part and then-part, a rule can be expressed as follows.

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; are the input variables of the fuzzy system; are the CFSs of the rule; are the input linguistic variables; are the parameters of CFS, which are the parameters of if-part; are the parameters of then-part. This CNFS can turn into a 6-layers neural network as Fig. 3, each layer will be described below.

**Layer 0**: This is the input layer. Inputs data are the results of multi-target feature selection. The time of the input vector is marked as follows.

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**Layer 1**: This is CFS layer, through the structure learning, several fuzzy sets can be constructed in different dimensions, and the input in each dimension can obtain membership degree via CFSs. Through the CFSs, the multiple complex-valued membership degrees can be obtained. Different membership degree can be applied to different model outputs for multi-target prediction. Through the formulas of complex fuzzy sets (2)-(9), a membership degree vector can be obtained as follows.

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**Layer 2**: This is if-part (premise) layer. Through the structure learning, we can select neurons, since the output of neuron is the product of each input, it is called neuron. Output of neuron is firing strength. And this study use the CFSs, input of each neuron is vector, the output is the same.

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, is the product of , is the membership degree of the membership degree vector in the neuron, .

**Layer 3**: This is normalization layer, the function is to normalize each element in input vector, and the output is a vector as follows.

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where, is the normalized value of the element in the neuron, . This study uses CFSs, the inputs are complex-valued and the outputs are the same.

**Layer 4:** This is then-part (consequence) layer, through the calculation in this layer, outputs can be obtained as follows.

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, is the output of the T–S neuron; {} are the parameters of the T–S neuron.

**Layer 5:** This is output layer, the sum of neuron outputs as the model output.

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Layer0

(Input layer)

Layer1

(CFS layer)

Layer2

(Premise layer)

Layer4

(Consequence layer)

Layer5

(Output layer)

Layer3

(Normalization layer)

Fig. 3. Complex neuro-fuzzy system.

## Parameter Learning

According to the concept of divide and conquer, we will use different machine learning algorithms to optimize the parameters of each layer to make it easier to find the best solution. For the parameter optimization of the first layer (CFS layer), we use two different algorithms, including particle swarm optimization (PSO) [19] and artificial bee colony optimization (ABCO) [17]. There are no parameters need to be optimized in the second layer and the third layer. In the fourth layer, the recursive least squares estimation (RLSE) [16] is used to optimize the parameters of consequence part. We expect that the combination of different algorithms will reduce the parameter dimensions of the search and bring better performance to the model. The details of three algorithms and the formulas are described below.

1. Particle Swarm Optimization

PSO algorithm is an evolutionary computational technique developed by J. Kennedy et al. [19] in 1995, derived from the simulation of society model. The principle is similar to the birds looking for food. In addition to the information provided by particle itself, inertia and its own best position, it also applies the group best position to adjust the speed as Fig. 4. The characteristics of this algorithm are fast convergence, and the formula is as follows.

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where, is the particle position in the iteration; is the particle velocity in the iteration; is the particle personal best position in the iteration; is the group best position in the iteration, are the parameters of PSO, are random number between 0 to 1. In this study, particle position means the parameters of the if-part, including the center, standard deviation and phase frequency parameter in each dimension.

Old position

New position

Adjustment by Pbest

Adjustment by Gbest

Group best position

(Gbest)

Personal best position

(Pbest)

Inertia direction

The particle

Fig. 4. PSO particle position update.

1. Artificial Bee Colony Optimization

The artificial bee colony algorithm was proposed by the Karaboga [17], and the principle is similar to the concept of bees searching for food sources. Its characteristics include swarm intelligence and randomness. For example, bees communicate about food source by a swing dance, and the swing dance includes the bias and randomness. In ABCO, there are three types of bees, including employed bee, onlooker bee, and scout bee. Where, employed bees seek the food source and dance to deliver the information, each time the communication has randomness, it represents the search for the overall dimension; The onlooker bee is responsible for searching near food sources. Firstly, use the roulette method to select one of the food sources and search around the food source, means parameters optimization for small dimension; The role of the scout bee is that when the food source has passed a certain number of developments, if the food source is not improved, the scout bee will replace the food source, which means randomly explore a new food source, avoiding the algorithm falling into local minimum. The process of ABCO is not the same as that bees seeking food in real world, the algorithm steps are as follows.

1. Randomly select an employed bee and form a new position which is food source, as follows.

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where, it the dimension of the food source; is the dimension of the employed bee; is the dimension of another random employed bee.

1. Onlooker bee uses roulette method to select one food source, where the better food source income, the easier it is to be selected. Roulette probability is as follows.

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where, is the probability of being selected for the food source; is the income, this study uses the reciprocal of the cost as the income; is the number of food sources.

1. Send all onlooker bees to search around the selected food source, formula is as follows.

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where, is the dimension of the onlooker bee; is the dimension of the selected food source; is the dimension of the other random food source, if the onlooker bee position is better than food source position, replace the food source position.

1. If an employed bee has reached the limit iteration and has no update, send the scout bee to replace it, formula of the scout bee position form is as follows.

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where, is the dimension of the food source; is the maximum of the dimension of the all food sources; is the minimum of the dimension of the all food sources.

1. Repeat steps 2~4 until the iterations end.
2. Recursive Least Squares Estimation

This study uses the Recursive least squares estimation (RLSE) [16] to update the T–S neuron parameters. The RLSE method uses each piece of data when updating parameters, and the recursive update is more effective than the LSE method that receives all the data at a time. In general LSE problems can be considered as a linear problem, as follows.

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where, is the target; is the model output; {} is the u known equations; {, =1,2,…,m} is the unknown parameter we estimate; is the model error. The LSE problem can also be expressed in the form of a matrix, as follows.

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is the input matrix, is the estimated unknown parameter matrix, is the target matrix, is the error matrix. It can optimize through the RLSE equations.

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where, is the number of recursive, {}, is the number of data, is the row of , when the RLSE algorithm start, is set to 0, is set to **,**  is a very big positive value, is an identity matrix.

In the hybrid algorithm, different part of the parameter optimization is responsible for different algorithms. This study use PSO-RLSE and ABCO-RLSE hybrid algorithm, PSO and ABCO is for if-part parameter optimization, RLSE is for parameter learning of the linear T-S function. Hybrid algorithm and model computation process are as follows.

1. Prepare the training data and testing data.
2. Use the particle position as the fuzzy set parameters. Input the training data into the model and calculate the firing strength of neuron.
3. Use RLSE to update the parameters of T-S neurons, and in RLSE equations are as follows.

|  |  |
| --- | --- |
| , |  |
| , |  |
| , |  |
| , |  |

where, . In the multi-target prediction, the normalized firing strength is a vector, which makes as a matrix. Therefore, we use identity matrix to replace the constant 1 in the equation (36), the improved equation is as follows.

|  |  |
| --- | --- |
| , |  |

1. After update all parameters, calculate the model output.
2. Calculate the cost, update the particle position in if-part and the relative data.
3. Repeat the steps 2~5 until iterations end.

## Investment Stratgy

In order to assess whether the model is helpful to the investment, the use of the cost function is not enough, because it is impossible to see whether there is profit, only the matching rate of the model can be understood, and the high matching rate does not mean that the investment efficiency is high. Therefore, this study uses the predicted closing price to combine the investment strategy [37] to decide buying or selling, the formulas are as follows.

|  |  |
| --- | --- |
| Buy: if  , |  |
| Sell: if  , |  |

where, is the threshold parameter, means the fluctuation of the stock; is the model output, means the predicted closing price; is the closing price. If predicted value is higher than the today closing value, means buying is the best choice, vice versa.

In order to make the decision more cautious, this study proposes another strategy as follows.

1. Use equations (43)-(44) to decide buying or selling.
2. Calculate the mean of past 30 days before the day as the second phase standard. Note that, after many test, the mean is best over the past 30 days.
3. If and then buying; if , and , then selling.
4. All operations have to pass two phase evaluation, if any one of the phase failed, it will not operate.

The profit calculation is made by the actual closing price today and the actual closing price next day as follows.

|  |  |
| --- | --- |
|  |  |
|  |

where, is actual profit, is the number of days which decision is buying; is the number of days which decision is selling; is the actual closing price of the day.

Through the above investment strategy and profit equations, we can get the profits made by model, and roughly simulate the effect of applying this model to the real world. This study will show profits from these two different strategies and compare with other literatures. In order to make the profit estimation more authentic, this study proposes the sliding window method to calculate the profit, which contains the concept of holding stocks. The steps are as follows.

1. Initialize window size, and each window means a closing date. If the window size is 10, means selling all hold stocks every 10 days.
2. Use the proposed strategy to decide operation.
3. If decision is buying, the hold stock number plus 1, and use principal to deduct the price of the day; if decision is selling, check whether hold stock is not 0, then sell all holding stocks at the price of the day and add the profit to the principal.
4. Repeat the steps 2~3, whenever the set window size is reached, sell all holding stocks, if data is smaller than window size, then sell all holding stocks at the last day of the data.

# Experimentation

There are three experiments in this study. The first experiment is a single target prediction. The target is the Taiwan stock exchange capitalization weighted stock index (TAIEX) in 2001 year; experiment 2 has two targets. The model is tested for the feasibility of the complex-valued membership degree. The first complex-valued output is used to predict the two targets, the real part is for the first target TAIEX, the imaginary part is for the second target Hang Seng index (HSI); experiment 3 is a multi-target prediction for four targets. The two complex-valued outputs are used, which are more complicated than Experiment 2. It includes the 2001 TAIEX and the Dow Jones industrial average index (DJI), national association of securities dealers automated quotation (NASDAQ) and standard & Poor's 500 (S&P 500). The above targets are very well-known stock indicators, for example, TAIEX is a weighted calculated indicator of listed stocks in Taiwan, which represents fluctuations of listed stocks in Taiwan; HSI is an important indicator reflecting the Hong Kong stock market. The index is calculated from the market capitalization of 50 HSI constituent stocks, which is equivalent to 63% of the 12-month average market capitalization rate of all listed companies on the Hong Kong Stock Exchange; DJI covers nine major industries such as finance, which is a stock price weighted indicator; NASDAQ is a market weighted indicator of more than 3,000 stocks, mostly in the technology industry; The S&P500 is the market capitalization of the top 500 US companies, including 11 industries such as IT. These indicators represent the trend of a country or even global stocks, so accurate prediction can bring a lot of help to investors.

In order to compare the performance of the model with other literatures, we will evaluate the model through the error indicators and calculate the profit after the virtual investment. Both the cost function and the evaluation index use the root mean square error (RMSE), and the formula is as follows.

|  |  |
| --- | --- |
| , |  |
| , |  |

, is the number of data; is the error vector of the data in the model; is the target; is the output vector of the data; is Hermitian transpose, means that after the matrix is transposed, the matrix elements are conjugated.

In the part of structure learning, the number of rules for all experiments in this study which also are the number of the neurons in the second layer, the upper limit is set to 15, and the lower limit is set to 4, so the number of neurons in the second layer of the model will be between 4 and 15.

In terms of simulation investment, the threshold of all the investment strategies of the experiment will be between 0 and 0.1, because the threshold parameter represents the fluctuation of the stock, while the rise and fall limit of the Taiwan stock is 10%, so the best will be selected from this interval. This study set 0.001 as the iterated step, and the search the threshold from 0 to 0.1. All experiments will find the best profit threshold through the training data, and calculate the profit of the testing data. If the profit in the training phase is 0, then testing data profit is set to 0, that is, it does not participate in the investment.

## Example 1—Time Series of Daily TAIEX

This experiment uses real-world time series data to verify the performance of the model, the target is TAIEX. The goal of the experiment is to establish an appropriate model that is trained to predict the daily stock price index. The model output is complex-valued, and the real part is used to predict. The data used is the daily closing price of TAIEX in 2001. The original data of this experiment was 278. After making first difference, 30 features were extracted from the data. Each feature has 247 data and the first 181 data are used as training data, the rest is used as testing data. After multi-target feature selection, the features are selected as model input data. The part of structure learning is clustered by the SC algorithm which is mentioned in the selection III, as described in the section III-B, there are {3, 3, 3, 3} complex fuzzy sets for each input dimension of the experiment. After the block selection, the original 144 π neurons were screened to the 15 π neurons. The model is properly constructed by the data drive concept. Each complex fuzzy set has three parameters, including the center, standard deviation, and phase frequency parameters. There are 12 complex fuzzy sets in total, so the number of parameters in the if-part is 36. The then-part type is T-S function, so there will be a total of parameters, is the number of then-parts, and is the number of model input dimensions. The parameters of the overall model after structure learning, as shown by TABLE I. Machine learning parameter settings, as shown in TABLE II. The results of this example are compared with the methods proposed in other papers [37], such as Chen, Yu, SVR, and ANFIS, comparison is shown as TABLE III. In order to test the stability of the model, we ran ten trials of experiments, and the performance statistics are shown in TABLE IV. The virtual investment profit is shown as TABLE V, it can be found that the lower standard deviation of the investment strategy proposed in this study, which means the lower the investment risk, and the average of the profit still is positive. The profit comparison of the simulated investment is shown in TABLE VI, and the profit from the model in this study is highest. The profit of the sliding window calculation method is shown in TABLE VII. The results of the target and model output are shown in Fig. 5; the machine learning curve of the model is shown in Fig. 6; the prediction errors are shown in Fig. 7.

TABLE I

Model setting (Experiment 1)

|  |  |
| --- | --- |
| **Parameter** | **Value** |
| Feature variables as model inputs | {} |
| Number of input fuzzy sets | {3, 3, 3, 3} |
| Type of fuzzy sets | CFS |
| Number of complex-valued targets\* | 1 |
| Number of neurons | 15 |
| Number of parameters in the CFS layer | 36 |
| Number of T–S neurons | 15 |
| Number of parameters in the T–S layer | 75 |

\* Each complex-valued target whose real and imaginary parts contain two real-valued targets, respectively.

TABLE II

Machine Learning setting

|  |  |
| --- | --- |
| **PSO** | |
| Swarm size | 50 |
| Iterations | 100 |
|  | {0.8, 2.0, 2.0} |
|  | Random in [0,1] |
| Initial particle positions | By SC algorithm in section III-B |
| Initial particle velocities | 0 |
| **ABCO** |  |
| Number of employed bees | 40 |
| Number of onlooker bees | 10 |
| Iterations | 100 |
| Limit | 20 |
| **RLSE** | |
|  |  |
|  | 25-by-1 zero vector |
|  | **I** |
| **I** | 25-by-25identify matrix |

TABLE IV

Ten Trials Performance (Experiment 1)

|  |  |  |
| --- | --- | --- |
| Trials | RMSE | |
| PSO-RLSE | ABCO-RLSE |
| 1 | 102.33 | 102.94 |
| 2 | 102.17 | 105.22 |
| 3 | 104.88 | 102.69 |
| 4 | 102.69 | 102.75 |
| 5 | 102.96 | **101.93** |
| 6 | **102.01** | 102.97 |
| 7 | 103.17 | 103.15 |
| 8 | 117.92 | 103.01 |
| 9 | 102.54 | 104.63 |
| 10 | 103.96 | 106.87 |

TABLE III

Performance Comparison (TAIEX, Experiment 1)

|  |  |
| --- | --- |
| Methods | RMSE |
| Chen [37] | 167 |
| Yu [37] | 148 |
| AR(1) [37] | 115 |
| SVR [37] | 114 |
| ANFIS [37] | 120 |
| Wei [37] | 110 |
| PSO-RLSE (proposed) | 102.01 |
| ABCO-RLSE (proposed) | **101.93** |

TABLE V

Virtual Investment Profits (Experiment 1)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Trials | PSO-RLSE | | ABCO-RLSE | | PSO-RLSE\* | | ABCO-RLSE\* | |
| Profit | | Profit | | Profit | | Profit | |
| Best | 0.052 | | 0.027 | | 0.057 | | 0.048 | |
| 1 | **990.76** |  | -430.77 |  | 0 |  | 0 |  |
| 2 | 698.90 |  | -484.46 |  | 102.42 |  | -25.62 |  |
| 3 | -1.7452 |  | -700.05 |  | **205.99** |  | 0 |  |
| 4 | -826.35 |  | **790.05** |  | -86.05 |  | 0 |  |
| 5 | 235.74 |  | 666.28 |  | 0 |  | **187.81** |  |
| 6 | 283.94 |  | 490.75 |  | 102.01 |  | 108.23 |  |
| 7 | -356.82 |  | 207.13 |  | 161.61 |  | 12.12 |  |
| 8 | 465.34 |  | 530.23 |  | -21.53 |  | 82.03 |  |
| 9 | 152.48 |  | -382.58 |  | 12.12 |  | -21.53 |  |
| 10 | -713.04 |  | 145.88 |  | 188.41 |  | 80.18 |  |
| Average | -81.43 | | 83.24 | | 66.49 | | 42.32 | |
| Std | 826.24 | | 541.78 | | 99.29 | | 69.52 | |
| Maximum | 990.76 | | 790.05 | | 205.99 | | 187.81 | |
| Minimum | -826.35 | | -700.05 | | -86.05 | | -21.53 | |

\*The result of the proposed investment strategy.

TABLE VI

Virtual Profit Comparison (Experiment 1)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chen [37] | Yu [37] | AR(1) [37] | SVR [37] | ANFIS [37] | Wei [37] | PSO-RLSE | ABCO-RLSE |
| Best | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.052 | 0.027 |
| Profit (TAIEX) | -92 | -73 | 671 | 202 | 686 | 795 | **990.76** | 790.05 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| TABLE VII  Sliding Window Profit (Experiment 1) | | | | | | | | | |
| Trials | PSO-RLSE | | | |  | ABCO-RLSE | | | |
| Window size | 5 | 10 | 20 | 30 |  | 5 | 10 | 20 | 30 |
| Best | 0.019 | 0.056 | 0.001 | 0.056 |  | 0.048 | 0.009 | 0.048 | 0.048 |
| 1 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |  | **177.02** | **269.14** | **704.83** | **269.14** |
| 6 | 102.01 | **161.69** | **161.69** | 9.18 |  | 82.03 | 0 | 609.84 | 174.15 |
| 7 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |  | 82.03 | 0 | 609.84 | 174.15 |
| 9 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | **319.26** |  | 0 | 0 | 0 | 0 |
| Average | 10.2 | 90.96 | 16.17 | 32.84 |  | 34.11 | 26.91 | 192.45 | 61.74 |
| Std | 32.26 | 148.68 | 51.13 | 100.67 |  | 60.7 | 85.11 | 310.95 | 102.72 |
| Maximum | 102.01 | 161.69 | 161.7 | 319.26 |  | 177.02 | 269.14 | 704.83 | 269.14 |
| Minimum | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |



(a) TAIEX Forecasting (PSO-RLSE)



(b) TAIEX Forecasting (ABCO-RLSE)

Fig. 5. Forecasting result. The blue line is the target, the red dash line is the model output. The result has great performance in the training and testing phase.



(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 6. Model forecasting errors. The errors in both two algorithms are in a random number range which is from -50 to 50, indicating that the prediction ability of the model is stable.

(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 7. Learning curve. (Experiment 1)

## Example 2—Double Time Series of Daily TAIEX and HSI

This experiment predicts two targets at the same time, namely TAIEX and HSI. The goal of the experiment is to establish an appropriate model that is trained to predict the daily stock price index. The model output is complex-valued, the real part is used to predict TAIEX and imaginary part is used to predict HSI. The data used is the daily closing price of TAIEX and HSI in 2000. The original data of this experiment was 277. After making first difference, we can get 276 data and the 30 features are extracted from each set of data for a total of 60 features. Each feature has 247 data and the first 204 data are used as training data, the rest is used as testing data. The first to the 30th features are TAIEX data, and the 31th to the 60th features are HSI. After multi-target feature selection, the features are selected as model input data. The part of structure learning is clustered by the SC algorithm which is mentioned in the selection III, as described in the section III-B, there are {3, 3, 3, 3} complex fuzzy sets for each input dimension of the experiment. After the block selection, the original 81 π neurons were screened to the 9 π neurons. The model is properly constructed by the data drive concept. Each complex fuzzy set has three parameters, including the center, standard deviation, and phase frequency parameters. There are 12 complex fuzzy sets in total, so the number of parameters in the if-part is 36. The then-part type is T-S function, so there will be a total of parameters, is the number of then-parts, and is the number of model input dimensions. The parameters of the overall model after structure learning, as shown by TABLE VIII. Machine learning parameter settings, as shown in TABLE IX. The results of this example are compared with the methods proposed in other papers [6], such as Chen, Yu, SR+ANFIS etc., comparison is shown as TABLE X. In order to test the stability of the model, we ran ten trials of experiments, and the performance statistics are shown in TABLE XI. The virtual investment profit is shown as TABLE XII, it can be found that the lower standard deviation of the investment strategy proposed in this study, which means the lower the investment risk, and the average of the profit still is positive. The profit comparison of the simulated investment is shown in TABLE XIII, and the profit from the model in this study is highest. The profit of the sliding window calculation method is shown in TABLE XIV. The results of the target and model output are shown in Fig. 8; the machine learning curve of the model is shown in Fig. 9; the prediction errors are shown in Fig. 10.

TABLE VIII

Model Setting (Experiment 2)

|  |  |
| --- | --- |
| **Parameter** | **Value** |
| Feature variables as model inputs | {} |
| Number of input fuzzy sets | {3, 3, 3, 3} |
| Type of fuzzy sets | CFS |
| Number of complex-valued targets\* | 1 |
| Number of neurons | 9 |
| Number of parameters in the CFS layer | 36 |
| Number of T–S neurons | 9 |
| Number of parameters in the T–S layer | 45 |

\* Each complex-valued target whose real and imaginary parts contain two real-valued targets, respectively.

TABLE IX

Machine Learning

|  |  |
| --- | --- |
| **PSO** | |
| Swarm size | 50 |
| Iterations | 100 |
|  | {0.8, 2.0, 2.0} |
|  | Random in [0,1] |
| Initial particle positions | By SC algorithm in section III-B |
| Initial particle velocities | 0 |
| **ABCO** | |
| Number of employed bees | 40 |
| Number of onlooker bees | 10 |
| Iterations | 100 |
| Limits | 20 |
| **RLSE** | |
|  |  |
|  | 25-by-1 zero vector |
|  | **I** |
| **I** | 25-by-25 identity matrix |

TABLE X

Performance Comparison (Experiment 2)

|  |  |  |
| --- | --- | --- |
| **Method** | **TAIEX** | **HSI** |
| Chen [6] | 413.27 | 280.15 |
| Yu [6] | 419.64 | 297.05 |
| SR+ANFIS [6] | 454.63 | 356.70 |
| SR+SVR [6] | 255.87 | 356.81 |
| Elman [6] | 154.21 | 302.27 |
| Cheng [6] | **150.55** | 251.70 |
| PSO-RLSE | 151.06 | 254.97 |
| ABCO-RLSE | 153.45 | **250.51** |

TABLE XI

Ten Trials Performance (Experiment 2)

|  |  |  |
| --- | --- | --- |
| Trials | PSO-RLSE | ABCO-RLSE |
| 1 | 298.49 | 296.49 |
| 2 | **296.36** | 304.76 |
| 3 | 302.48 | 296.26 |
| 4 | 298.20 | 296.25 |
| 5 | 305.08 | 296.59 |
| 6 | 332.39 | **293.77** |
| 7 | 308.42 | 314.39 |
| 8 | 303.56 | 295.34 |
| 9 | 297.36 | 297.58 |
| 10 | 329.41 | 307.30 |

TABLE XII

Virtual Investment Profit (Experiment 2)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Trials | PSO-RLSE | | ABCO-RLSE | | PSO-RLSE\* | | ABCO-RLSE\* | |
| Profit | | Profit | | Profit | | Profit | |
| Best | 0.029 | | 0.073 | | 0.047 | | 0.011 | |
| 1 | -127.26 |  | -1540.7 |  | 167.42 |  | 128.39 |  |
| 2 | -1712.9 |  | **2062.4** |  | 122.46 |  | 35.49 |  |
| 3 | -2726.6 |  | -1337 |  | 166.24 |  | -61.21 |  |
| 4 | -2022.7 |  | -822.64 |  | 62.53 |  | -143.9 |  |
| 5 | 909.16 |  | 814.5 |  | 515.98 |  | **285.72** |  |
| 6 | -969.99 |  | -1902.1 |  | 500.97 |  | -209.4 |  |
| 7 | 1453.9 |  | 163.68 |  | 25.05 |  | 149.63 |  |
| 8 | -2712.9 |  | 665.28 |  | 66.42 |  | -95.68 |  |
| 9 | **2515.3** |  | -1274 |  | **800.26** |  | 128.39 |  |
| 10 | -826.09 |  | -941.3 |  | -121.6 |  | 202.16 |  |
| Average | -622 | | -411.17 | | 230.57 | | 41.95 | |
| Std | 1790 | | 1276.3 | | 282.92 | | 162.95 | |
| Maximum | 2515.3 | | 2062.4 | | 800.26 | | 285.72 | |
| Minimum | -2726.6 | | -1902.1 | | -121.6 | | -209.4 | |

\*The result of the proposed investment strategy.

TABLE XIII

Virtual Investment Profit Comparison (Experiment 2)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chen [6] | Yu  [6] | SR+ANFIS [6] | SR+SVR [6] | Elman [6] | Cheng et al. [6] | PSO-RLSE | ABCO-RLSE |
| Best | 0.012 | 0.012 | 0.012 | 0.012 | 0.012 | 0.012 | 0.029 | 0.073 |
| Profit (TAIEX) | 0 | 0 | 0 | 0 | 0 | -231.02 | **1367.8** | 922.95 |
| Profit (HSI) | -1471 | -1368 | -602.94 | 190.71 | 2342 | 1793.12 | **2393.1** | 1139.4 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| TABLE XIV  Sliding Window Profit (Experiment 2) | | | | | | | | | |
| Trials | PSO-RLSE | | | |  | ABCO-RLSE | | | |
| Window size | 5 | 10 | 20 | 30 |  | 5 | 10 | 20 | 30 |
| Best | 0.023 | 0.024 | 0.098 | 0.015 |  | 0.001 | 0.001 | 0.002 | 0.002 |
| 1 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| Average | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| Std | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| Maximum | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |
| Minimum | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |



(a) TAIEX Forecasting (PSO-RLSE)

(b) TAIEX Forecasting (ABCO-RLSE)



(c) HSI Forecasting (ABCO-RLSE)



(d) HIS Forecasting (ABCO-RLSE)

Fig. 8. The model output. The blue line is the target, red dash line is the model output, it can be found the two algorithm have great performance.



(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 9. Model prediction errors. The errors in the both algorithm are in the random range, indicating the model prediction ability is stable.



(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 10. Learning Curve (Experiment 2)

PSO-RLSE is stable in the 75th iteration, and the ABCO-RLSE is stable in the 25th iteration.

## Example 3—Quadruple Time Series of Daily TAIEX, DJI, NASDAQ and S&P500

This experiment predicts four targets at the same time, namely TAIEX, DJI, NASDAQ, S&P500. The goal of the experiment is to establish an appropriate model that is trained to predict the daily stock price index. The model has two complex-valued outputs, the real part of the first output is used to predict TAIEX, the imaginary part of the first output is used to predict DJI, the real part of the second output is used to predict NASDAQ and the imaginary part of the second output is used to predict S&P500. The data used is the daily closing price of TAIEX, DJI, NASDAQ and S&P500 in 2001. The original data of this experiment was 278. After making first difference, we can get 277 data and the 30 features are extracted from each set of data for a total of 120 features. Each feature has 247 data and the first 181 data are used as training data, the rest is used as testing data. The first to the 30th features are TAIEX data, the 31th to the 60th features are HIS, the 61th to the 90th features are NADSDAQ data, and the 91th to the 120th features are S&P500. After multi-target feature selection, the features are selected as model input data. The part of structure learning is clustered by the SC algorithm which is mentioned in the selection III, as described in the section III-B, there are {3, 3, 3, 3} complex fuzzy sets for each input dimension of the experiment. After the block selection, the original 81 π neurons were screened to the 9 π neurons. The model is properly constructed by the data drive concept. Each complex fuzzy set has three parameters, including the center, standard deviation, and phase frequency parameters. There are 12 complex fuzzy sets in total, so the number of parameters in the if-part is 36. The then-part type is T-S function, so there will be a total of parameters, is the number of then-parts, and is the number of model input dimensions. The parameters of the overall model after structure learning, as shown by TABLE XV. Machine learning parameter settings, as shown in TABLE XVI. The results of this example are compared with the methods proposed in other papers [27], such as SVR, ANFIS, RBF and CNFS-ARIMA etc., comparison is shown as TABLE XVII. In order to test the stability of the model, we ran ten trials of experiments, and the performance statistics are shown in TABLE XVIII. The virtual investment profit is shown as TABLE XIX, it can be found that the lower standard deviation of the investment strategy proposed in this study, which means the lower the investment risk, and the average of the profit still is positive. The profit comparison of the simulated investment is shown in TABLE XX, and the profit from the model in this study is highest. The profit of the sliding window calculation method is shown in TABLE XXI. The results of the target and model output are shown in Fig. 11; the machine learning curve of the model is shown in Fig. 12; the prediction errors are shown in Fig. 13.

TABLE XV

Model Setting (Experiment 3)

|  |  |
| --- | --- |
| **Parameters** | **Value** |
| Feature variables as model inputs | {} |
| Number of input fuzzy sets | {3, 3, 3, 3} |
| Type of fuzzy sets | CFS |
| Number of complex–valued targets\* | 2 |
| Number of neurons | 9 |
| Number of parameters in the SCFS layer | 36 |
| Number of T–S neurons | 9 |
| Number of parameters in the T–S layer | 45 |

\* Each complex-valued target whose real and imaginary parts contain two real-valued targets, respectively.

TABLE XVI

Machine Learning

|  |  |
| --- | --- |
| **PSO** | |
| Swarm size | 50 |
| Iterations | 100 |
|  | {0.8, 2.0, 2.0} |
|  | Random in [0,1] |
| Initial particle positions | By SC algorithm in the section III-B |
| Initial particle velocities | 0 |
| **ABCO** |  |
| Number of employed bees | 40 |
| Number of onlooker bees | 10 |
| Iterations | 100 |
| Limits | 20 |
| **RLSE** | |
|  |  |
|  | 25-by-1 zero vector |
|  | **I** |
| **I** | 25-by-25 identity matrix |

TABLE XVII

Performance comparison (Experiment 3)

|  |  |  |
| --- | --- | --- |
| Method | TAIEX | DJI |
| SVR (two models, each with single output) [27] | 162.46 | 101.44 |
| ANFIS (two models, each with single output) [27] | 147.36 | 105.56 |
| ANFIS (one model with two outputs) [27] | 151.62 | 128.20 |
| RBF (two models, each with single output) [27] | 134.32 | 106.33 |
| RBF (one model with two outputs) [27] | 137.58 | 181.79 |
| CNFS(5)-ARIMA (one model with two outputs) [27] | 115.82 | 103.06 |
| PSO-RLSE | **101.61** | 101.82 |
| ABCO-RLSE | 102.61 | **100.79** |

TABLE XVIII

Ten Trial Performance Comparison (Experiment 3)

|  |  |  |
| --- | --- | --- |
| Trials | PSO-RLSE | ABCO-RLSE |
| 1 | 185.99 | 199.32 |
| 2 | 183.75 | 183.89 |
| 3 | 183.72 | 204.56 |
| 4 | 183.81 | 256.10 |
| 5 | 184.39 | **180.82** |
| 6 | 191.31 | 199.00 |
| 7 | **181.90** | 217.09 |
| 8 | 281.103 | 182.13 |
| 9 | 183.68 | 188.24 |
| 10 | 183.67 | 183.89 |

TABLE XIX

Virtual Investment Profit (Experiment 3)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Trials | PSO-RLSE | | ABCO-RLSE | | PSO-RLSE\* | | ABCO-RLSE\* | |
| Profit | | Profit | | Profit | | Profit | |
| Best | 0.049 | | 0.049 | | 0.059 | | 0.025 | |
| 1 | -444.90 |  | 942.69 |  | 341.79 |  | **951.12** |  |
| 2 | 615.12 |  | 1206 |  | 0 |  | 266.13 |  |
| 3 | 880.74 |  | -850.1 |  | 52.399 |  | -139.9 |  |
| 4 | -1629.7 |  | 67.36 |  | 0 |  | 352.73 |  |
| 5 | -365.89 |  | **3201.7** |  | 149.49 |  | 682.72 |  |
| 6 | 762.83 |  | 1374.9 |  | **740.06** |  | 141.16 |  |
| 7 | **2146.7** |  | 2156.9 |  | 80.34 |  | 149.49 |  |
| 8 | -805.41 |  | 784.34 |  | 310.11 |  | 410.97 |  |
| 9 | -1741.7 |  | 3460.7 |  | 0 |  | 324.16 |  |
| 10 | -1189.6 |  | 1206 |  | 0 |  | 266.13 |  |
| Average | -177.19 | | 1355.1 | | 167.42 | | 340.47 | |
| Std | 1252.4 | | 1314 | | 238.49 | | 300.64 | |
| Maximum | 2146.7 | | 3201.7 | | 740.06 | | 951.12 | |
| Minimum | -1741.7 | | -850.1 | | 0 | | -139.9 | |

\*The result of the proposed investment strategy.

TABLE XX

Virtual Investment Profit Comparison (Experiment 3)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Chen [37] | Yu [37] | AR(1) [37] | SVR [37] | ANFIS [37] | Wei [37] | PSO-RLSE | ABCO-RLSE |
| Best | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.049 | 0.049 |
| Profit (TAIEX) | -92 | -73 | 671 | 202 | 686 | 795 | 1104.6 | **1409.9** |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| TABLE XXI  Sliding Window Profit (Experiment 3) | | | | | | | | | |
| Trials | PSO-RLSE | | | |  | ABCO-RLSE | | | |
| Window size | 5 | 10 | 20 | 30 |  | 5 | 10 | 20 | 30 |
| Best | 0.022 | 0.022 | 0.017 | 0.064 |  | 0.006 | 0.006 | 0.05 | 0.022 |
| 1 | **457.97** | **1225.6** | 325.31 | 140.61 |  | 0 | 0 | 1837.7 | 602.43 |
| 2 | 0 | 0 | 0 | 0 |  | 0 | 0 | 1724.5 | 658.46 |
| 3 | 354.04 | 538.28 | **1409.7** | **1159.8** |  | 177.02 | 269.14 | 1760.4 | 453.38 |
| 4 | 0 | 0 | 0 | 0 |  | 48.85 | 48.85 | 1704.2 | 397.15 |
| 5 | 0 | 0 | 0 | 107.53 |  | 0 | 0 | **2292.7** | 618.23 |
| 6 | 280.44 | 955.11 | 325.31 | 364.55 |  | 412.09 | 471.78 | 471.78 | 619.26 |
| 7 | 150.82 | 242.94 | 678.63 | 496.28 |  | 0 | 0 | 119.82 | 0 |
| 8 | 0 | 0 | 0 | 0 |  | 410.98 | **1760.3** | 650.62 | 0 |
| 9 | 0 | 0 | 0 | 0 |  | **766.13** | 1010.1 | 1881.4 | **857.54** |
| 10 | 0 | 0 | 0 | 0 |  | 0 | 0 | 1724.5 | 658.46 |
| Average | 124.33 | **296.19** | 273.89 | 226.88 |  | 181.51 | 356.01 | **1416.8** | 456.49 |
| Std | 176.98 | 457.74 | 460.58 | 371.34 |  | 264.57 | 591.50 | 723.62 | 284.61 |
| Maximum | 457.97 | 1225.6 | 1409.7 | 1159.8 |  | 766.13 | 1760.3 | 2292.7 | 857.54 |
| Minimum | 0 | 0 | 0 | 0 |  | 0 | 0 | 119.82 | 0 |



(a) TAIEX Forecasting (PSO-RLSE)



(b) TAIEX Forecasting (ABCO-RLSE)



(c) DJI Forecasting (PSO-RLSE)



(d) DJI Forecasting (ABCO-RLSE)



(e) NASDAQ Forecasting (PSO-RLSE)



(f) NASDAQ Forecasting (ABCO-RLSE)



(g) S&P500 Forecasting (PSO-RLSE)



(h) S&P500 Forecasting (ABCO-RLSE)

Fig. 11. Model forecasting. The blue line is the target, red dash line is the model output, it can be found the two algorithm have great performance.



(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 12. Prediction errors. The errors are in a random number range which is from -100 to 100, indicating that the prediction ability of the model is stable. (Experiment 3)



(a) PSO-RLSE



(b) ABCO-RLSE

Fig. 13. Learning curve. The RMSE variation can be seen from this curve. PSO-RLSE is stable in the 60th iteration, and the ABCO-RLSE is stable in the 75th iteration.

# Discussion

This study proposes to optimize the model parameters with the hybrid algorithm PSO-RLSE and ABCO-RLSE, and use the divide and conquer principle to reduce the search loading of the algorithm. When the model constructing, the model size is controlled by machine learning through the concept of data driven. Before the training data enters the model, the feature selection is performed in advance, and the feature data that is more favorable for the prediction target is selected for time series prediction. According to the 3 experiments, it is verified that the series of methods proposed in this study have better prediction performance for the prediction of time series.

Data preprocessing of this study uses multi-target feature selection [28], taken from Shannon information entropy [34]. The information entropy and the information entropy difference between the data are used to calculate the amount of information that the data candidate feature can provide for the target data, and to consider the amount of redundant information caused by the selected feature. Finally, from a large amount of data, the feature data that is useful for single target or multiple targets is selected to reduce the burden and computational complexity caused by too much data. Structure learning will determine the size of the model. Firstly, the initial position of the complex fuzzy set is preliminarily determined by the SC algorithm. Finally, the concept of data density is introduced, a smaller number of blocks are selected from a large number of blocks as the rules for reducing the burden of the model. From the 3 experiments, it can be found that the original number of blocks is reduced from 144, 81, 81 to 15, 9, and 9, which obviously reduces the burden on the model calculation. The establishment of the number of rules will also affect the number of the consequence layer. Therefore, the effective reduction of the number of rules will also affect the number of parameters that need to be learned during parameter learning, making it easier to find the best solution in the parameter search process.

The CNFS is established by CFSs and T-S fuzzy systems. Through the nonlinear Gaussian CFSs and the linear T-S fuzzy system combined into a nonlinear fuzzy system, the one-to-one relationship of the neurons is generated by the If-Then rule, so that the system can be understood by humans. Compared with the traditional fuzzy set, the CFS extends from the one-dimensional real space to the planar space of a two-dimensional unit disk. This method enables the membership degree to accommodate more information, which helps to improve the reasoning ability and application performance of the fuzzy system. Using the complex-valued membership degree makes the model system can generate a set of complex-valued outputs, so that it can simultaneously predict two real targets. Through the deconstruction of the membership degree, 3 complex-valued membership degree can be obtained, so that the original 3 outputs can predict 6 targets, means the prediction ability of the model is enhanced.

In the parameter learning of the CNFS, the if-part uses PSO and ABCO to learn, both of which have the concept of using swarm intelligence. Through the information exchange in the group find a better location. PSO has 3 characteristics. First, the automatic pace adjustment will adjust the position through weight and swarm intelligence. Second, randomness, the component with random variables when updating velocity, contributes to particle activity; third, stability, each time when updating the position and velocity, its moving direction is according to the best solution of the group and its best solution; ABCO also has 3 characteristics. First, randomness, onlooker bee uses roulette method when selecting the food source, so the selected food source has a randomness; Second, local search, Through the onlooker bees searching near the food source, it can avoid missing the best solution. Third, the jump mechanism can judge the food source effect by whether there is any update. If the food source is not updated within certain iterations, then a scout bee will replace the food source location, reducing the probability of being trapped in a local minimum. The then-part uses the RLSE algorithm to find a linear function by using the input data points and the previous calculation results, so that the square error of the data points and the function reaches a minimum value, thereby finding a function that conforms to the data. To optimize the parameters of the consequence part by continuous recursive calculations.

Through a series of methods, it can be seen that the 3 experimental results of this study are better than other methods, indicating that the model has good prediction ability in single target, double target and multi-target. In the 3 experiments, it can be found that the ABCO-RLSE hybrid algorithm performs better than the PSO-RLSE, means local search and tripping mechanisms has a certain effect, making the model output more accurate. In the virtual investment, we have an interesting finding that the performance of the model is not positively related to the virtual investment profit. When a model performs well, it does not mean that the virtual investment will be good. That is, in the 10 model trials, the trial of best performance is not the same as the simulation investment profit best trial, because the curve fitting of the model represents the matching rate, but it does not mean that it can effectively earn profits. Buying and selling are the critical factors. The operation of buying and selling is determined by the threshold parameter in the investment strategy. This parameter will affect the profit. Therefore, it can be found the best is different in PSO-RLSE and ABCO-RLSE in TABLE VI, TABLE XIII and TABLE XX, it is different from other literatures too. This study has a better performance compared with the past literature, but the predicting of the stock market's ups and downs are still difficult. In the new investment strategy proposed in this study, the past rise and fall values can be used to set another threshold in an objective manner, making investment more cautious. As expected, the standard deviation from TABLE V, TABLE XII and TABLE XIX can be found to be lower than the previous investment strategy, which means that the stability of investment profit is higher, and the average value is higher, indicating that the probability of making money is high. The sliding window profit calculation method is closer to the real world transaction. From 3 experiments, it can be found that different models and different data have different window size, but most of them have a best window size of not less than 10.

# Concluding Remarks

In this study, a new hybrid algorithm ABCO-RLSE is proposed, which combines the artificial bee colony algorithm and the recursive least squares method to optimize the parameter set of the CNFS. The model combines a complex fuzzy set, a T-S model system, and a neural network. Before the data enters the model, the feature selection is to avoid redundant input data entering the model, which decreases computational efficiency and increases the burden of the model. The complex fuzzy set makes the model have multiple complex-valued outputs. Through the deconstruction of the complex-valued membership degree, the real part and the imaginary part of the membership degree can take out other membership degrees, so that the model can generate three complex-valued outputs and predict up to six real targets, making the model different from the traditional fuzzy system and capable of simultaneous multi-target prediction. Experiment 2 and Experiment 3 demonstrate the contribution of this approach. In terms of structure learning, the model does reduce the size effectively, and through the concept of data density, select the blocks that are more useful for the data, objectively use the data to build the model, avoid complex calculations and improve the efficiency of the model. When parameters learning, ABCO can find the best solution more easily than PSO because of the jump mechanism and local search characteristics. The performance in three experiments is better than the methods proposed in other literatures. The investment strategy proposed in this study has a large chance of making money through the cooperation of threshold value. The standard deviation of the 10 simulated investment profits is lower than that of the investment strategy proposed in other literatures, which helps to reduce the risk of investors and provides the objective information through the data to investors.

Through the experiments in this study, we can find that the performance of the model is better than other literature methods, which proves the optimization ability of PSO and ABCO. However, these two algorithms still have their own limitations. For example, PSO may be premature, which means that the model converges in the first few iterations, which makes the model easy to fall into the local minimum; ABCO is limited to the local minimum when searching. The onlooker bee will search near the food source, when the food source update effect is not good, the overall effect will be bad, and the calculation time required is also more longer than the PSO algorithm. Therefore, other algorithms can be used in the future, combined with CNFS to try to find better performance. In terms of investment strategy, we find that the best threshold parameters are different for different models. Therefore, in the future, we can optimize the threshold parameters through machine learning algorithms, or adjust the weight of the investment according to the predicted rise and fall for more precise investment operations and making profits from the stock market.

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